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The structure group revisited

Following the treatment of a class of quasi-linear SPDE with Sauer, Smith, and Weber, we approach Hairer's regularity structure $(\mathsf{A}, \mathsf{T}, \mathsf{G})$ from a different angle. In this approach, the model space T is a direct sum over an index set that corresponds to specific linear combination of (decorated) trees, and thus amounts to a more parsimonious parameterization of the solution manifold. Moreover, the same structure group G captures different classes of equations; depending on the class, different (sub)spaces T matter, which correspond to linear combinations of different types of trees.

In our approach to G, we start from the space of tuples (a, p) of (polynomial) nonlinearities a and space-time polynomials p, which we think of parameterizing the entire manifold of solutions u (satisfying the equation up to space-time polynomials) via re-centering. We consider the actions of a shift by a space-time vector $h \in \mathbb{R}^{d+1}$ and of tilt by space-time polynomial q on (a, p)-space, where, crucially, the tilt by a constant is treated as a shift of the (one-dimensional) u-space. We consider the infinitesimal generators of these actions, and pull them back as derivations on the algebra of formal power series $\mathbb{R}[[\mathsf{z}_k, \mathsf{z}_{\mathbf{n}}]]$ in the natural coordinates $\{\mathsf{z}_k\}_{k\in\mathbb{N}_0}$ and $\{\mathsf{z}_{\mathbf{n}}\}_{\mathbf{n}\in\mathbb{N}_0^{d+1}-\{\mathbf{0}\}}$ of (a, p)-space. This defines a Lie algebra $\mathsf{L} \subset \mathrm{Der}(\mathbb{R}[[\mathsf{z}_k, \mathsf{z}_{\mathbf{n}}]])$. Loosely speaking, the corresponding Lie group coincides with G^*, but we follow an algebraic path to construct G.

As a group, $\mathsf{G} \subset (\mathsf{T}^+)^*$ arises in the standard way from the Hopf algebra T^+ that is obtained from dualizing the universal enveloping algebra $\mathrm{U}(\mathsf{L})$. Here, gradedness and finiteness properties are needed for the well-posedness of the co-product $\Delta^+\colon \mathsf{T}^+ \to \mathsf{T}^+ \otimes \mathsf{T}^+$ and the antipode. The passage from $\mathbb{R}[[\mathsf{z}_k, \mathsf{z}_{\mathbf{n}}]]$ to a smaller (linear) subspace T^* is needed for dualizing the module defined through $\mathsf{L} \subset \mathrm{End}(\mathsf{T}^*)$ to obtain the co-module $\Delta\colon \mathsf{T} \to \mathsf{T}^+ \otimes \mathsf{T}$. This yields the representation $\mathsf{G} \subset \mathrm{End}(\mathsf{T})$. Both Δ and Δ^+ satisfy the postulates of regularity structures, in particular the properties that intertwine Δ, Δ^+, and the family of re-centering maps $\mathcal{J}_{\mathbf{n}}\colon\ \mathsf{T} \to \mathsf{T}^+$. The latter relies on choosing a natural basis of $\mathrm{U}(\mathsf{L})$, different from the standard Poincaré-Birkhoff-Witt basis, for dualization.

This is joint work with P. Linares and M. Tempelmayr.