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Unified Signature Cumulants and Generalized Magnus Expansions

The signature of a path can be described as its full non-commutative exponential. Following T. Lyons we regard its expectation, the expected signature, as path space analogue of the classical moment generating function. The logarithm thereof, taken in the tensor algebra, defines the signature cumulant. We establish a universal functional relation in a general semimartingale context. Our work exhibits the importance of Magnus expansions in the algorithmic problem of computing expected signature cumulants, and further offers a far-reaching generalization of recent results on characteristic exponents dubbed diamond and cumulant expansions; with motivation ranging from financial mathematics to statistical physics. From an affine process perspective, the functional relation may be interpreted as infinite-dimensional, non-commutative ("Hausdorff") variation of Riccati's equation. Many examples are given.

This is a joint work with Peter K. Friz and Paul Hager (`arXiv:2102.03345`).
